Some statistical inversion approaches for simultaneous thermal diffusivity and heat source mapping

by H. Massard da Fonseca*, H. R. B. Orlande*, O. Fudym**,

*Dept.of Mechanical Engineering, Politécnica/COPPE, Federal University of Rio de Janeiro, UFRJ, Cid. Universitaria, Cx. Postal: 68503 Rio de Janeiro, RJ, 21941-972, Brazil, hmassard@enstimac.fr, helcio@mecanica.ufrj.br
**Université de Toulouse; Mines Albi; CNRS; Centre RAPSODEE, Campus Jarlard, F-81013 Albi cedex 09, France, olivier.fudym@mines-albi.fr

Abstract

The mapping of thermophysical properties from thermal images provided by an infrared camera is a difficult inverse problem, due to the large amount of data to be processed and large number of parameters to be estimated, as well as the low signal-to-noise ratio. It is thus of great interest to implement estimation approaches of low computational cost that can accurately cope with such inherent difficulties. In this paper, we make use of the so-called statistical inversion approach for the solution of an inverse problem that involves the identification of both local thermal diffusivity and local source term.

1. Introduction

The estimation of spatially varying thermophysical properties from thermal images provided by infrared cameras is a quite involved inverse problem, mainly due to the large amount of measured data to be processed under a low signal-to-noise ratio and the large number of parameters to be estimated. For such reasons, it is thus of interest to implement a linear estimation technique, in order to reduce the associated computational costs and to obtain the inverse problem solution within a feasible time.

The minimization of a prediction error model instead of the traditional output error model [1] has been successfully used for such purpose, basically by the following two different approaches that rely on the so-called nodal strategy, namely: (i) The use of local estimation algorithms based on the correlations between pixels, which allow for direct inversion [2-6]; (ii) The use of techniques within the Bayesian framework [7,8,9].

Unfortunately, both approaches require that the sensitivity matrix be computed through linear operations on the measured data. Then, the stochastic nature of the regression matrix may yield a bias in the estimates when an ordinary least squares approach is computed [2]. On the other hand, the use of techniques within the Bayesian framework allows that prior information on the unknown quantities be taken into account, including the stochastic sensitivity matrix [7,8,9].

In the Bayesian approach to statistics, an attempt is made to utilize all available information in order to reduce the amount of uncertainty present in an inferential or decision-making problem. As new information is obtained, it is combined with previous information to form the basis for statistical procedures. The formal mechanism used to combine the new information with the previously available information is known as Bayes’ theorem [7-24]. In reference [7], the nodal strategy was used in conjunction with a Markov Chain Monte Carlo (MCMC) method for the estimation of the spatially distributed thermal diffusivity in a one-dimensional heat conduction problem. This approach was then extended in [8] for the simultaneous estimation of the spatially distributed thermal diffusivity and heat source term in a in a two-dimensional heat conduction problem. However, for both cases, the unknown quantities were constant in time. In [9] the estimation of a transient heat source term that also varies spatially was presented, in a two-dimensional linear heat conduction problem in a thin plate, such as the one considered in [8]. Such state estimation problem was solved with the Kalman filter [10,13,24-31].

In this paper, experimental results are shown for the estimation of constant thermal diffusivity and source term, which is the same as the theoretical problem presented in [8]. The experiments were carried out by heating a sample and measuring the resulting temperature increase with an infrared camera. The sample was heated with an electrical resistance placed in the back face of the sample, and the temperature was measured in the front face of the sample.

2. Physical problem and mathematical formulation

The physical problem examined in this paper involves two-dimensional transient heat conduction in a plate, with spatially varying thermal conductivity and volumetric heat capacity. Boundaries are supposed insulated. This situation can be found when testing a thin layer medium with partial lumping across the plate. For thick samples, the possible separation of in-plane and out-of-plane temperature distribution has been discussed in [32]. Both internal steady heat generation and convective heat losses are also considered. The initial temperature within the medium is non-uniform due to the initial randomly distributed photo-thermal heat pulse. The mathematical formulation for this problem is given by:


\[ C(x, y) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[ k(x, y) \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[ k(x, y) \frac{\partial T}{\partial y} \right] - h(x, y) (T - T_m) + g(x, y) \]

\[ \text{(1.a)} \]

in \(0 < x < L_x, 0 < y < L_y\), for \(t > 0\)

\[ \frac{\partial T}{\partial x} = 0 \quad \text{at } x = 0 \text{ and } x = L_x, \text{ for } t > 0 \]

\[ \text{(1.b)} \]

\[ \frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0 \text{ and } y = L_y, \text{ for } t > 0 \]

\[ \text{(1.c)} \]

\[ T(x, y, 0) = T_0(x, y) \quad \text{for } t = 0, \text{ in } 0 < x < L_x, 0 < y < L_y, \]

\[ \text{(1.d)} \]

Where \(C(x,y)\) is the local volumetric heat capacity in \(Jm^{-3}K^{-1}\), \(k(x,y)\) is the local thermal conductivity in \(Wm^{-1}K^{-1}\), \(h(x,y)\) is the local convective heat transfer coefficient in \(Wm^{-2}K^{-1}\) and \(g(x,y)\) is the local heat source, in \(Wm^{-3}\).

3. Nodal strategy and predictive model for the solution of the inverse problem

In this paper we deal with the solution of an inverse problem involving the identification thermophysical properties of a homogeneous medium, by using experimental data provided by an infrared camera. Such measurement technique is quite powerful because it can provide accurate non-intrusive measurements, with fine spatial resolutions and at large frequencies.

For the solution of the inverse problem, we assume that transient temperature measurements are available at several positions within the medium. For the application of the nodal strategy [3,32] we rewrite equation (1.a) in the following non-conservative form:

\[ \frac{\partial T}{\partial t} = a(x, y) \nabla^2 T + \frac{1}{C(x, y)} \left[ \frac{\partial k(x, y)}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial k(x, y)}{\partial y} \frac{\partial T}{\partial y} \right] - H(x, y) (T - T_m) + G(x, y) \]

\[ \text{(2)} \]

\[ a(x, y) = \frac{k(x, y)}{C(x, y)} \quad H(x, y) = \frac{h(x, y)}{C(x, y)} \quad G(x, y) = \frac{g(x, y)}{C(x, y)} \]

An explicit discretization of equation (2) using finite-differences results in:

\[ Y_{i,j}^{n+1} = L_{i,j}^n a_{i,j} + D_{x,i,j} \delta_{i+1,j}^x + D_{y,i,j} \delta_{i,j+1}^y - \Delta t (T_{i,j}^{n} - T_m) H_{i,j} + \Delta t G_{i,j} \]

\[ \text{(3)} \]

where the subscripts pair \((i,j)\) denotes the finite-difference node at \(x_i = i \Delta x, i = 1 \ldots n_x\) and \(y_j = j \Delta y, j = 1 \ldots n_y\) and the superscript \(n\) denotes the time \(t = nt, n = 0 \ldots (n-1)\). The other quantities appearing in equation (3) are given by:

\[ Y_{i,j}^{n+1} = T_{i,j}^{n+1} - T_{i,j}^{n} \]

\[ \text{(4.a)} \]

\[ L_{i,j}^n = \Delta t \left( \frac{T_{i+1,j}^{n} - 2T_{i,j}^{n} + T_{i-1,j}^{n}}{\Delta x^2} + \frac{T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{\Delta y^2} \right) \]

\[ \text{(4.b)} \]

\[ D_{x,i,j} = \frac{\Delta t}{2 \Delta x} (T_{i+1,j}^{n} - T_{i-1,j}^{n}) \]

\[ \text{(4.c)} \]

\[ D_{y,i,j} = \frac{\Delta t}{2 \Delta y} (T_{i,j+1}^{n} - T_{i,j-1}^{n}) \]

\[ \text{(4.d)} \]

\[ \delta_{i,j}^x = \frac{1}{C(x, y)} \frac{\partial k}{\partial x} \quad \text{and} \quad \delta_{i,j}^y = \frac{1}{C(x, y)} \frac{\partial k}{\partial y} \]

\[ \text{(4.e,f)} \]
Equation (4.a) defines the forward temperature difference in time. Equation (4.b) approximates the Laplacian of temperature at time \( t \) and node \((i,j)\). Equations (4.c-d) are relative to the spatial derivatives of temperature.

By writing equation (3) for a given node \((i,j)\) and all time-steps, we obtain:

\[
Y_{i,j} = J_{i,j} P_{i,j}
\]

where \( J_{i,j} = \begin{bmatrix}
L_{1,i,j}^x & D_{x,i,j} & D_{y,i,j} & -\Delta t(T_{i,j}^1 - T_{i,j}^\infty) & \Delta t \\
L_{2,i,j}^x & D_{x,i,j} & D_{y,i,j} & -\Delta t(T_{i,j}^2 - T_{i,j}^\infty) & \Delta t \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
L_{i,j}^x & D_{x,i,j} & D_{y,i,j} & -\Delta t(T_{i,j}^N - T_{i,j}^\infty) & \Delta t
\end{bmatrix}
\]  

(6.a)

and \( Y_{i,j} = \begin{bmatrix}
Y_{i,j}^1 \\
Y_{i,j}^2 \\
\vdots \\
Y_{i,j}^N
\end{bmatrix} \) and \( P_{i,j} = \begin{bmatrix}
a_{i,j} \\
\delta_{i,j}^x \\
\delta_{i,j}^y \\
H_{i,j} \\
G_{i,j}
\end{bmatrix} \)  

(6.b-c)

The vector of parameters at node \((i,j)\) such as defined by equation (6.c) contains the following parameters:

- \( a_{i,j} \), which is the local thermal diffusivity in \( m^2.s^{-1} \).
- \( \delta_{i,j}^x \) and \( \delta_{i,j}^y \), which are the local thermal conductivity gradients along the x and y directions, respectively, divided by the heat capacity.
- \( H_{i,j} \) is the local convective coefficient divided by the local heat capacity, similar to a velocity in \( m.s^{-1} \).
- \( G_{i,j} \) is the local heat source divided by the local heat capacity, similar to a heating rate in \( K.s^{-1} \).

The vector of parameters at node \((i,j)\) contains five parameters involving only four independent functions \( k \), \( C \), \( h \) and \( g \). Moreover, all the five parameters appear in Eq.(6.c) and divided by \( C \). This fact makes the simultaneous estimation of the five parameters quite ill-conditioned. However, for materials with sharp interfaces as in the cases examined herein, the thermal conductivity gradients are zero except at the interfaces. Moreover, as it was shown by the sensitivity analysis [7-9], the sensitivity coefficient with respect to the convective heat transfer coefficient is very low for the cases of interest for this work, so that this parameter can be neglected in the formulation of the problem. Also, by considering the thermal diffusivity \( a = k/C \) as an unknown parameter to be estimated, instead of the thermal conductivity \( k \), turns out that the two remaining parameters \( a \) and \( G \) are independent.

The sensitivity matrix defined in equation (6.a) is a function of the temperature field, thus depends on these parameters. This fact would yield a non-linear estimation procedure when minimizing an output error model, that is based on the difference between the direct model given by equation (5) and the corresponding experimental data.

One way to circumvent this problem is to use a predictive error model, such as shown in figure (1). In that scheme, the measurement data are directly passed to the model, seen as a predictor.

\[ e(t) \]

\[ \hat{Y}(t) \]

*Fig.1. Minimization of the predictive error model*

The objective function to minimize may be chosen as the \( L_2 \) norm of the predictive error \( e(t) \), such as
\[ S_{ij} = e_{ij}^T e_{ij} = (Y_{ij} - J^T P) (Y_{ij} - J^T P) \]  

(7)

In equation (7), the measurements in \( Y_{ij} \) are chosen as the “observable data” \( (y_{mi}(t) \) in Fig.1), while the predictive model, obtained from the sensitivity matrix, is also built with the experimental data \( \hat{Y}(t) \) in Fig. 1.

Equation (5), when used as a predictor, yields a linear dependence of the system response with respect to the vector of parameters, based on the knowledge of the sensitivity matrix. With the spatial resolution and frequency of measurements made available by infrared cameras, the sensitivity matrix can be approximately computed with the measurements \([2,3]\). It is of interest to point out that the choice is made to solve simultaneously as many local linear measurements; the measurements \( Y_{ij} \) are omitted in order to simplify the notation, but the estimation still is made locally at each node.

4. Bayesian approach for the solution of inverse problems

In the Bayesian approach to statistics, an attempt is made to utilize all available information in order to reduce the amount of uncertainty present in an inferential or decision-making problem. As new information is obtained, it is combined with any previous information to form the basis for statistical procedures. The formal mechanism used to combine the new information with the previously available information is known as Bayes’ theorem \([7-24]\). Therefore, the term Bayesian is often used to describe the so-called statistical inversion approach, which is based on the following principles \([13]\):

1. All variables included in the model are modeled as random variables.
2. The randomness describes the degree of information concerning their realizations.
3. The degree of information concerning these values is coded in probability distributions.
4. The solution of the inverse problem is the posterior probability distribution.

Bayes’ theorem can then be stated as \([7-24]\):

\[
\pi_{\text{posterior}}(P) = \pi(P|Y) = \frac{\pi_{\text{prior}}(P) \pi(Y|P)}{\pi(Y)} \tag{8}
\]

where \( \pi_{\text{posterior}}(P) \) is the posterior probability density, that is, the conditional probability of the parameters \( P \) given the measurements \( Y \); \( \pi_{\text{prior}}(P) \) is the prior density, that is, the coded information about the parameters prior to the measurements; \( \pi(Y|P) \) is the likelihood function, which expresses the likelihood of different measurement outcomes \( Y \) with \( P \) given; and \( \pi(Y) \) is the marginal probability density of the measurements, which plays the role of a normalizing constant.

If we assume the parameters and the measurement errors to be independent Gaussian random variables, with known means and covariance matrices, and that the measurement errors are additive, a closed form expression can be derived for the posterior probability density. In this case, the likelihood function can be expressed as \([7-24]\):

\[
\pi(Y|P) = \left(2\pi\right)^{-M/2} \left| W^{-1} \right|^{-1/2} \exp \left[-\frac{1}{2} (Y - Y^*)^T W (Y - Y^*) \right] \tag{9}
\]

where \( Y^* \) is the vector of estimated variables, obtained from the solution of the forward model with an estimate for the parameters \( P \), \( M \) is the number of measurements and \( W \) is the inverse of the covariance matrix of the errors in \( Y \).

Similarly, for the case involving a prior normal distribution for the parameters we can write:

\[
\pi(P) = \left(2\pi\right)^{-Mp/2} \left| V \right|^{-1/2} \exp \left[-\frac{1}{2} (P - \mu)^T V^{-1} (P - \mu) \right] \tag{10}\]

where \( \mu \) and \( V \) are the known mean and covariance matrix for \( P \), respectively. By substituting equations (9) and (10) into Bayes’ theorem, except for the normalizing constant in the denominator we obtain:

\[
\ln \left[ \pi(P|Y) \right] \propto -\frac{1}{2} (MN) \ln 2\pi + \ln \left| W^{-1} \right| + \ln \left| V \right| + S_{MAP}(P) \tag{11}
\]

where \( S_{MAP}(P) = (Y - T(P))^T W (Y - T(P)) + (\mu - P)^T V^{-1} (\mu - P) \)  

(12)

Equation (11) reveals that the maximization of the posterior distribution function can obtained with the minimization of the objective function given by equation (12), denoted as the \textit{maximum a posteriori} objective function \([4,5,9-11]\). Equation (12) clearly shows the contributions of the likelihood and of the prior distributions in the objective function, given by the first and second terms on the right-hand side, respectively.
For linear estimation problems such as the one considered here, the minimization of the maximum a posteriori objective function is obtained with [11]:

$$
\hat{P}_{\text{MAP}} = [J^T W J + V^{-1}]^{-1} [J^T W Y + V^{-1} \mu]
$$

(13)

It is apparent in equation (13) how the prior information is used as a regularization term. Obviously, when the prior information for the parameters is poor, the diagonal terms of the covariance matrix are very large, and the MAP estimator tends towards the maximum likelihood estimator, and there is no regularization effect.

On the other hand, if different prior probability densities are assumed for the parameters, the Posterior Probability Distribution may not allow an analytical treatment. In this case, Markov Chain Monte Carlo (MCMC) methods are used to draw samples of all possible parameters, so that inference on the posterior probability becomes inference on the samples [13,14,17].

In order to implement the Markov Chain, a density \( q(P^*, P^{(t-1)}) \) is required, which gives the probability of moving from the current state in the chain \( P^{(t-1)} \) to a new state \( P^* \).

The Metropolis-Hastings algorithm [13,14,17] was used in this work to implement the MCMC method. It can be summarized in the following steps:

1. Sample a Candidate Point \( P^* \) from a jumping distribution \( q(P^*, P^{(t-1)}) \).
2. Calculate:

$$
\alpha = \min \left[ 1, \frac{\pi(P|Y) q(P^{(t-1)}, P^*)}{\pi(P^{(t-1)}|Y) q(P^*, P^{(t-1)})} \right]
$$

(14)

3. Generate a random value \( U \) which is uniformly distributed on (0,1).
4. If \( U \leq \alpha \), define \( P^t = P^* \); otherwise, define \( P^t = P^{(t-1)} \).
5. Return to step 1 in order to generate the sequence \( \{P^1, P^2, \ldots, P^n\} \).

In this way, we get a sequence that represents the posterior distribution and inference on this distribution is obtained from inference on the samples \( \{P^1, P^2, \ldots, P^n\} \). We note that values of \( P \) must be ignored until the chain has converged to equilibrium. For more details on theoretical aspects of the Metropolis-Hastings algorithm and MCMC methods, the reader should consult references [13,14,17].

In the nodal strategy described in section 3, the sensitivity matrix is approximately computed with the measurements. Therefore, for the implementation of the Metropolis-Hastings algorithm the uncertainties in the computation of the sensitivity matrix need to be taken into account. By assuming that \( P \) and \( J \) are independent random variables, the sought posterior probability density is then given by [13]:

$$
\pi(P,J|Y) \propto \pi(Y|P,J) \pi(P) \pi(J)
$$

(15)

where \( \pi(J) \) is the a priori distribution for the sensitivity matrix \( J \).

5. Experimental Apparatus

Fig. 1 Experimental scheme: (a) circular electrical resistance (b) samples, resistance and insulation

In this work, the experimental temperature measurements were made with the 560M Titanium infrared camera, from Cedip Infrared Systems. The electrical signal given by the camera is a function of the received photons in the sensor of the camera. This electrical signal is then converted to a Digital Level (DL). The transfer functions photons/volts and volts/DL are linear increasing functions. The camera’s detector has 640 x 512 pixels, and is sensitive in the spectrum between 1.5 and 5.0 µm. The sample is made in an epoxy plate, whose thermophysical properties were determined
experimentally by the Flash method, in the LFA 447/1 Nanoflash apparatus. For the resin used in this work, three tests were made at 20°C. The mean diffusivity retrieved with the LFA 447/1 was 0.24 mm²/s, with a standard deviation of 0.001 mm²/s.

6. Estimation results obtained from the experimental measurements

For the application of the technique developed in this paper, the sample was heated with a circular 25 Ω resistance with 23.5 mm in diameter, electrically insulated with Kapton tape, as shown in Figure 2a, with an effective area of about 2x10⁻⁶ m². In this way, the power dissipated by the resistor is approximately 1.6 W for the applied voltage of 10 V. The thickness of the sample used in this work is 0.0011 m and \( C = 1.833 \times 10^6 \text{ J/(Km)}^3 \), which yields \( G = 1.93 \text{ K/s} \), (see Eq. 2). The resistance was positioned at the bottom of the resin – see figure 2b.

The discretization of the plate is made with 98 nodes, and the values of the local parameters \( a_{i,j} \), \( H_{i,j} \) and \( G_{i,j} \) are being estimated for these nodes. The apparent spatial resolution for one pixel in the image is \( \Delta x = 76.9 \mu \text{m} \), and the corresponding observed part of the sample is thus \( L_x = L_y = 0.00754 \text{ m} \). The experimental time duration is 1.01s, with a time step \( \Delta t = 0.01 \text{ s} \), which was convenient for the explicit finite difference scheme stability criterion. In figure 3 are shown the initial temperature field and the location of some particular pixels chosen for plotting some specific results.

The prior distribution relative to the thermal diffusivity is assumed to be uniform in the range \([8 \times 10^{-8}, 9 \times 10^{-7}] \text{ m²/s} \). The prior information regarding \( G_{i,j} \) is assumed to be uniform in the range \([0, 5] \text{ K/(ms)} \), which is in correspondence with the heat flux range generated by the electrical resistance such as \([0, 10081.5] \text{ W/m²} \). The prior information on \( H_{i,j} \) is assumed to be uniform in the domain \([0.002, 0.2] \text{ 1/s} \), corresponding to \([4.03, 403] \text{ W/(m²°C)} \) for the convective coefficient \( h \). The Markov chain relative to each pixel was generated with 6000 states, while the first 1000 were discarded in the final statistical analysis (computation of statistical parameters). The acceptance rate of the Metropolis hastings algorithm was found to be around 60%, which is quite acceptable. The changing rate of the parameters was chosen to be 2% for \( a_{i,j} \) and \( G_{i,j} \), 0.2% for the sensitivity matrix, and 10% for \( H_{i,j} \).

The average results obtained for the estimation of thermal diffusivity are presented in table 1. As the thermal diffusivity is constant in the plate, all the estimates can be averaged. The standard deviation is also shown in table 1, where the MCMC algorithm is also compared to the Ordinary Least Squares estimation. The standard deviation obtained by the MCMC method is slightly lower than by OLS. Both are coherent with the reference value.

<table>
<thead>
<tr>
<th>Estimation method</th>
<th>Thermal diffusivity (mm²/s)</th>
<th>Standard Deviation (mm²/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference value</td>
<td>0.24</td>
<td>-</td>
</tr>
<tr>
<td>MCMC</td>
<td>0.253</td>
<td>0.045</td>
</tr>
<tr>
<td>OLS</td>
<td>0.259</td>
<td>0.08</td>
</tr>
</tbody>
</table>

In order to show how the local thermal diffusivity parameters are retrieved, in figure 4 are plotted the variation of this parameter along some specific lines, such as \((i = 17, i = 36, i = 64 \text{ and } i = 81)\), which correspond to the spatial lines positions at \((x = 0.00134 \text{ m}, x = 0.00277 \text{ m}, x = 0.00492 \text{ m} \text{ and } x = 0.00623 \text{ m})\). The results obtained from OLS and MCMC are plotted and compared with the reference value. For both methods, the estimated values present some
oscillations due to the noise effect around the reference value. MCMC results are shown to be slightly closer and smoother than OLS estimates.

**Fig. 4. Local thermal diffusivity estimation along some specific lines**

The MCMC approach not only yields a single average value, but the posterior distribution of all the estimated parameters – that is for each Markov chain. An example is shown in figure 5, where the posterior marginal distributions retrieved for the thermal diffusivity corresponding to two pixels such as ($i = 30$, $j = 90$) and ($i = 58$, $j = 59$), which are located at $(x = 0.00231$ m, $y = 0.00692$ m) and $(x = 0.00460$ m, $y = 0.00454$ m) are plotted in an histogram form. It can be observed in that figure that the posterior distributions follow a Gaussian distribution, obtained as expected due to the Gaussian likelihood and the uniform prior distribution assumed this work. The choice of a Gaussian prior was also tested in this work, yielding an identical result as for the uniform prior.

**Fig. Error! No hay texto con el estilo especificado en el documento.** Posterior distribution obtained for pixels (30,90) and (58,59)
The retrieved heat source map is drawn in figure 6. The MCMC estimation method is able to estimate the functional shape of $G$. It may also be observed in this figure that the magnitude of the estimated value for $G$ is close to the reference value, as given in section 5.

The estimated values of the heat source are plotted on some selected lines in figure 7, in order to check the functional form of the retrieved profile, for the lines of pixels $i = 17$, $i = 36$, $i = 64$ and $i = 81$ which correspond to the lines located at $x = 0.00131$ m, $x = 0.00277$ m, $x = 0.00492$ m and $x = 0.00623$ respectively. The estimated values are found to be in quite good accordance with the reference value.

In figure 8 are plotted the residual curves, which are the temperature difference between the experimental signal and the model computed with the estimated parameters, for both the OLS and the MCMC methods. For the sake of clarity, only a few curves for some particular pixels are given herein. Four pixels are considered such as: $(i = 38, j = 40)$, $(i = 64, j = 60)$, $(i = 30, j = 90)$ and $(i = 58, j = 59)$ which are located at $(x = 0.00292$ m, $y = 0.00308$ m), $(x = 0.00492$ m, $y = 0.00461$ m), $(x = 0.00231$ m, $y = 0.00692$ m) and $(x = 0.00446$ m, $y = 0.00454$ m) respectively. It is to be noticed that the MCMC method yields smoother residuals than OLS, which confirms the lower uncertainty in the retrieved parameters obtained by adding some prior information to the estimation process. Moreover, the relative difference
between the experimental and predicted temperatures remain lower than 1%, and are poorly correlated, which seems to confirm the validity of the model.

Fig. 8. Residual curves shown for four particular pixels: temperature difference between the experimental signal and the model computed with the estimated parameters.

7. Conclusions

This work was aimed at the experimental determination of local thermophysical properties and source terms constant in time. The solution of the inverse problem has been rewritten to apply to the nodal strategy. Temperature measurements were made with the infrared camera. The MCMC method allowed to obtain better and less dispersed results than the Ordinary Least Square method. Moreover, this kind of approach yields the posterior distribution of the retrieved parameters. The computational cost of this method is however the price to pay for this information. For the present work, the computational time for 6000 states was about 40 mn, while it was more or less one second for OLS. Future works include the estimation of transient magnitudes.

8. Acknowledgements

The financial support provided by FAPERJ, CAPES and CNPq, Brazilian agencies for the fostering of science, is greatly appreciated. The authors also acknowledge in France the support of CNRS and MAE. This work was partially funded by the STIC AmSud project « I3PE – Inverse Problems in Physical Properties Estimation ».

REFERENCES


