

Fiber orientation assessment in complex shaped parts reinforced with carbon fiber

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Abstract

The use of composite materials is growing more and more every day in several applications. The arrangement or orientation of the fibers relative to one another have a significant influence on the strength and other properties of fiber reinforced composites. Thus, evaluation techniques are needed for measuring material fiber orientation. In this work Infrared Thermography is employed to assess the material's fiber orientation. More specifically a pulsed infrared diode laser heating spot technique is used in order to assess fiber orientation on the surface of complex shaped parts made of carbon/PEEK (*Polyether ether ketone*) randomly-oriented strands (ROS).

1. Introduction

The use of composite materials (CM) has grown during the past years and this growth will continue in the near future. Today they are widely used in industry. One of the reasons is because their strength and stiffness are comparable to metals with the added advantage of significant weight reduction. Efficient, fast and low-cost manufacturing technologies are required as a large number of structures will be manufactured using composites in the near future. Also, technologies to measure and insure the quality of those manufactured parts are (and will be) in high demand. The arrangement or orientation of the fibers relative to one another, concentration and distribution all have a significant influence on the strength and stiffness of fiber reinforced composites. Thus, one needs to develop testing techniques to assess the material's fiber orientation. Destructive methods can be employed to evaluate the fiber on a composite, e.g. cutting a section of the material, polishing the area and evaluating it with microscopy. However, the destructive approach is not always an option since the sample will be damaged after the inspection and probably unfit for use. Thus, Non-Destructive Testing and Evaluation (NDT&E) techniques must be employed in some cases to assess the material's fiber content.

Infrared Thermography (IT) is a well-known and widely used NDT&E technique for composite material inspection. In active IT an external heat source is used to stimulate the material being inspected in order to generate a thermal contrast between the feature of interest and the background. The active approach is adopted in many cases given that the inspected parts are usually in equilibrium with the surroundings [1]. Qualitative and quantitative assessment of flaws is a very popular application of IT for CM. However there are others such as fiber orientation assessment. In this paper IT is used in order to assess fiber orientation of composite materials on the surface of complex shaped parts made of carbon/PEEK (*Polyether ether ketone*) Random-Oriented Strands (ROS). A "laser spot" technique, referred to here as Pulsed Thermal Ellipsometry (PTE), is used to locally measure the material's fiber orientation. In the case of fiber reinforced composites, which are thermally anisotropic, the thermal pattern formed on the surface of the material due to the thermal stimulation is an ellipse where its major axis orientation is related to the fiber orientation.

Figure 1 shows the part tested in this paper (with curvatures, edges and corners). In the case of ROS parts, fiber orientation on the surface is random since each strand has its own fiber orientation. Due to this ROS characteristic, several strands (located in different regions) were tested using PTE. For a more accurate quantitative result, mapping of infrared data onto a 3D scanned part (or dense 3D point cloud) was applied to aide in the correct quantitative interpretation of the thermal pattern on the part's surface and as a consequence provide a more accurate fiber orientation measure. It is worth mentioning here that only one camera was used and it was kept motionless during all tests (i.e.



Figure 1 - Complex shaped part moulded with random-oriented strands (ROS)

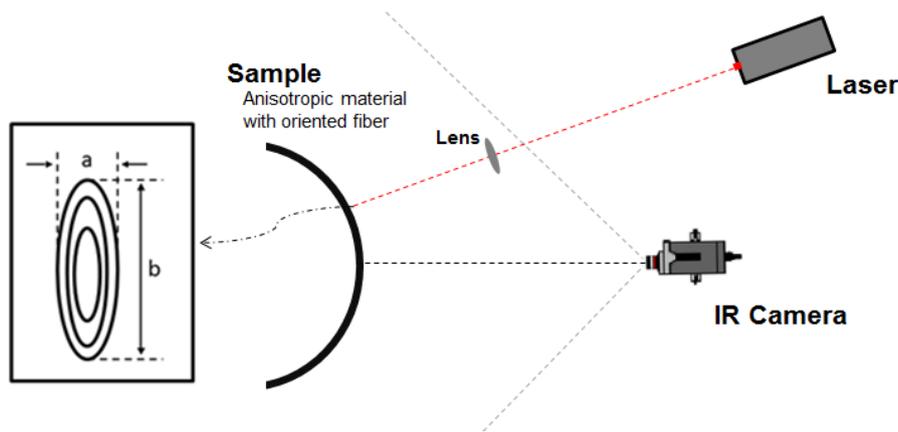


Figure 2 - Pulsed Thermal Ellipsometry (PTE) schematic set-up for fiber orientation assessment in composite materials

scene view and camera's angle of incidence did not change according to the tested region and where the region was on the part's surface).

This paper is organized as follows: the next section presents a brief literature review on IT for fiber orientation assessment (PTE); in Sec. 3 the used 3D approach is described; in Sec. 4, experiments and results are presented and discussed; and our final considerations are presented in Sec. 5.

2. Pulsed Thermal Ellipsometry

More than one century ago, De Senarmont [2] applied a thermal approach to determine the principal orientations in crystal plates: he covered them with a thin layer of wax, heated them over a small spot and monitored the isotherm shape revealed by the solid/liquid transition contour appearing in the wax layer. The isotherm proved to be elliptical and its aspect ratio is related to the square root of the principal conductivities in the surface plane.

This method, referred to by Krapez et al. [3, 4, 5, 6] as *Thermal Ellipsometry*, was later used for various applications (with, of course, up-to-date experimental equipment) by means of thermography. It was applied on polymer materials to establish a correlation between their draw ratio and the induced thermal anisotropy. It was also used to evaluate the fiber orientation in the case of composite materials using short or long carbon fibers. For the latter problem, authors such as Aindow et al. [7] and Cielo et al. [8] showed that heat propagates faster in the longitudinal direction of fiber on the surface of fiber reinforced laminates. Aindow et al. [7] detected local anisotropy in carbon fiber-reinforced plastic (CFRP - nylon-66) injection mouldings by two methods: thermography using infrared scanning, which reveals anisotropy of thermal conductivity, and polarized shear-wave ultrasonic showing elastic anisotropy. For the thermographic method, they recorded isotherms formed around a point source of heat on a plane surface using an infrared imaging camera. The isotherms that they observed were ellipses of which the ratio of lengths of the principal axes (b/a) are proportional to the square root of the ratio of thermal conductivities. They assumed that the longest dimension of the counter in each picture indicated the major axis of thermal conductivity in the surface, which in turn is related to the direction of fiber orientation.

Cielo et al. prepared a review [8] of a number of optical techniques for the characterization of non-metallic materials. One possibility reported by them is the evaluation of phase (or fiber) orientation in stretched polymer films or in composites by an analysis of the thermal propagation pattern. A typical configuration is shown in figure 2. They spot-heated the inspected part by a narrow laser beam and the resulting heat-propagation pattern was recorded by an IR camera. If the material is oriented, such as the unidirectional graphite-epoxy sheet they inspected, an elliptical thermal pattern is observed, with the ratio between the two principal axes (b/a) being related to the square root of the thermal conductivities in the longitudinal and transverse directions. A test on an isotropic material would give a circle instead of an ellipse. They illustrated this approach showing results from two 8-ply unidirectional *Narmco 5217* sheets spot heated for a period of 20 seconds by a laser beam of 0.5 W.

A more detailed theoretical analysis was later undertaken by Krapez [3] through an analytical treatment of thermal diffusion in laminates composed of orthotropic layers assuming the surface is submitted to concentrated heating. Three temporal regimes were considered in that study: steady-state regime, transient regime (as obtained during step heating), and modulated regime (in order to analyze how the so-called thermal waves "propagate" in orthotropic laminates). Experiments were performed on carbon-epoxy laminates for all three regimes. Later, Krapez [4] used the same theory (thermal anisotropy measurements method which consists in analyzing the shape of the isotherms which develop around a heated spot) to develop a thermal inversion method to infer thickness of skin and core layers of a 3-layer carbon/epoxy laminate.

Karpen et al. [9] used lock-in thermography (harmonic thermal waves) to probe orientation fields of carbon fibers both along the surface and in depth at low modulation frequencies and within a short time. Later they developed a theoretical model in order to correctly interpret their measurements [10].

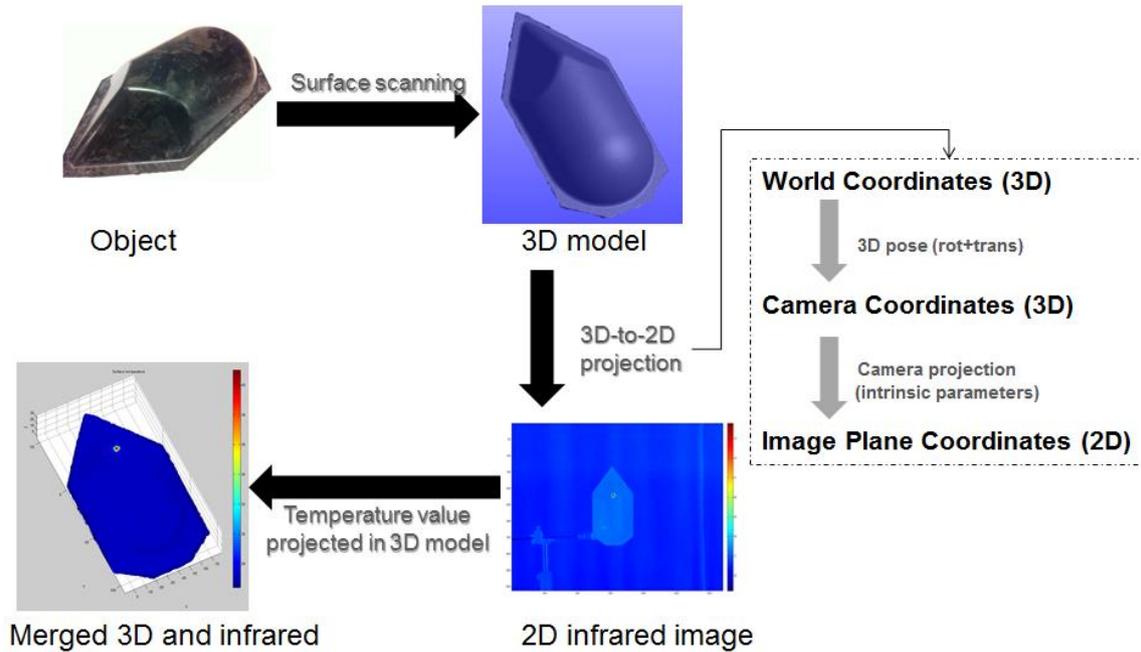


Figure 3 – Steps of fusion process of infrared and 3D data

3. Fusion of infrared and 3D data

The thermal pattern produced due to the heating spot applied on the part is formed on its surface. Since the inspected part has a complex shape (not a flat surface) the quantitative interpretation of the ellipse will be distorted if it is visualized directly in the raw 2D image since it is not possible to retrieve depth information from the 2D image (at least directly). The surface point temperature information would be projected on a 2D plane and the geometry information of the 3D surface would be lost. It would result in a wrong measurement of the fiber orientation direction of that spot on the surface. To overcome this problem, a 3D model of the part is used together with the thermal image to ensure a correct quantitative analysis of the ellipse and consequently correct measure of fiber orientation on the surface.

The infrared information provided by the infrared camera is combined with a 3D model of the surface of the part (or a 3D dense point cloud) which is obtained by previously scanning the part. The merge of these two pieces of information (infrared and 3D) allows the correct quantitative analysis of the elliptical pattern overcoming the problem of losing the surface's geometric information and also the case where the camera does not have a frontal view of the inspected area (i.e. camera's angle of incidence greater than 0°). It was shown in a previous paper [11] that in flat laminates the image capture by the infrared camera is still suitable for the fiber orientation measurement up to a certain camera's angle of incidence.

Figure 3 summarizes the basic steps necessary to fuse the infrared and 3D data. In this section it is presented how the 3D data is combined with the 2D infrared image.

3.1. Capturing the 3D model of the part

To capture the 3D shape of objects a commercially available 3D scanner (Creaform Go!SCAN[®] [12]) has been used. In contrast to laser scanners the scanner and the software (Creaform VXelements[®]) allow the user to build a whole 3D model of an object by automatically combining multiple measurements. The handheld scanner has to be manually moved around the object to do this. The scanner is geometrically calibrated and reports the object dimensions in millimeters. The model resolution can be as small as 0.5 mm. The software guides the user during acquisition and post-processing of the data. It offers post-processing functionalities like surface reconstruction/optimization, hole filling and artifacts removal. Also, features of interest can be introduced: points, lines, planes and surface can be fitted in the model. Points of interest previously selected are added to the model to be used in the next steps of the 3D/infrared merging process. The generated mesh, with all its added features, can be exported in various ASCII and binary standard formats. The exported data is imported into Matlab[®] to perform the next steps of the 3D/infrared fusion. By default the center of the used coordinate system is the position and orientation of the scanner in the first scan. However the software provides options to change the coordinate system and align it with points, planes or lines in the 3D model. Figure 5.a. shows the final 3D model generated with Creaform VXelements[®] with 5 selected points which are going to be used to find the 3D pose of the object in the infrared image. Figure 5.b. shows the same points selected in the infrared 2D image.

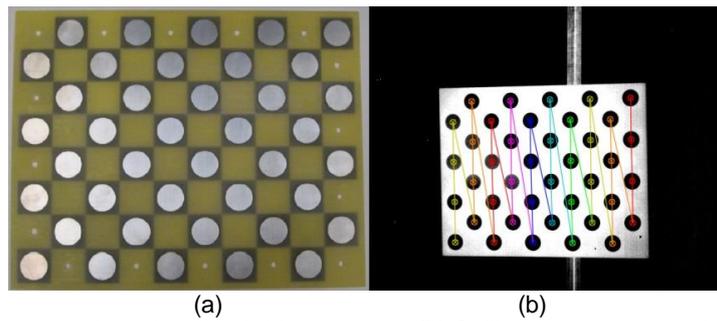


Figure 4 – (a) Calibration target viewed in the visible spectrum, (b) Calibration target seen in the infrared spectrum with the detected features.

3.2. Infrared camera calibration

In order to correctly merge the 3D and infrared information, it is important to know the camera parameters. To determine these camera parameters one needs to perform a camera calibration. The calibration process consists in identifying the internal quantities of the camera that affect the imaging process: image center position (in pixels), focal length, different scaling factors for row pixels and column pixels, skew factor and lens distortion. These parameters describe the relationship of the 3D model coordinate system and the camera coordinate system [13]. During the calibration process, the camera observes a set of features such as points or lines with known positions in some fixed world coordinate system. In this context, camera calibration can be modeled as an optimization process, where the discrepancy between the observed image features and their theoretical position is minimized with respect to the camera's intrinsic parameters.

A target with circle patterns is used in this project (similar to the ones found in [14, 15]). Features in the target must be seen by the infrared camera thus before image acquisition, the target is heated so the circles can be detected. Figure 4 shows the calibration target and the features detected during the calibration process in the infrared image. Several packages are available, already implemented e.g. in OpenCV or in Matlab[®] [16], to perform this task. Camera pixels are assumed to be square so scaling factors for row and column pixels are neglected as well as the skew factor. The other intrinsic parameters, focal length, image center and lens distortion, are stored and used in the camera perspective projection step.

3.3. From 3D world coordinates to 2D image pixel coordinates

3.3.1. 3D pose

The next step required to combine the 3D model and the infrared image is to find the object's 3D pose. It consists in identifying a rotation and translation matrix that maps 3D world points (in 3D model reference system) in 3D camera points (in the camera reference system). A popular algorithm called "POSIT" (*pose from orthography and scaling with iterations*) [17] is used. It is a fast iterative algorithm for determining the pose (rotation and translation) of an object or scene with respect to a camera when points of the object are given in some object coordinate system and these points

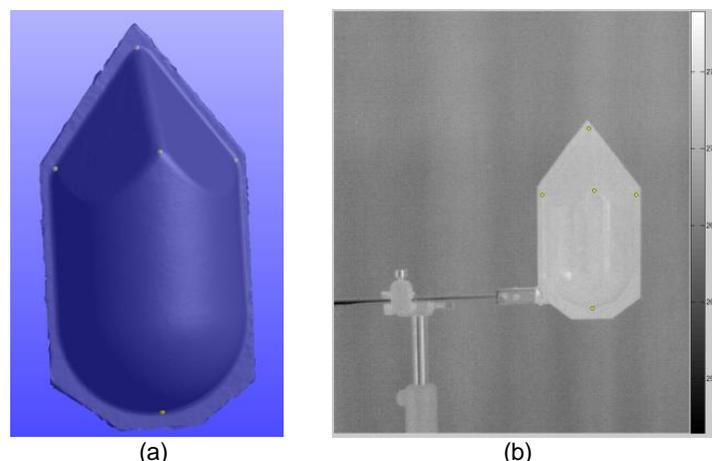


Figure 5 – (a) 3D generated model with 5 selected points shown in gray. (b) 2D infrared image with the same 5 selected points shown in yellow.

are visible in the camera image and recognizable, so that corresponding image points and object points can be listed in the same order. POSIT requires at least 4 points to be recognized in the 3D model and in the 2D image. The points selected here were the 5 points shown in Figure 5. Implementation of POSIT can be found online in the author's webpage [18]. The algorithm determines rotation and translation matrixes after 3 iterations.

The transformation from world coordinates (3D model reference system) to camera coordinates can be written as [13]:

$$\begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix} = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_W \\ Y_W \\ Z_W \\ 1 \end{bmatrix} \quad (1)$$

where $P_W = (X_W, Y_W, Z_W, 1)$ is the point in homogenous coordinates in the world reference system, $P_C = (X_C, Y_C, Z_C, 1)$ is the point in homogenous coordinates in the camera reference system, R is a 3x3 rotation matrix and T a 3x1 translation matrix.

3.3.2. Camera perspective projection

After finding the 3D point coordinate in the camera's reference system by applying the rotation and translation calculated with the POSIT algorithm, the object must be projected onto the image plane in order to determine the correspondence between the 3D points and the pixels in the infrared 2D image. The coordinates (u, v) of the pixels p in the infrared image are given by [13]:

$$\begin{cases} u = \frac{x'}{z'} \\ v = \frac{y'}{z'} \end{cases}, \quad (2)$$

where:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} f & 0 & c_x & 0 \\ 0 & f & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X_C \\ Y_C \\ Z_C \\ 1 \end{bmatrix}, \quad (3)$$

f is the camera focal length and (c_x, c_y) are the coordinates of the image center position in pixel coordinates.

At the end of this process, a relation between a point in the world is created with a pixel of the infrared image ($P_W = (X_W, Y_W, Z_W)$ and $p = (u, v)$). Therefore, the temperature value of each point in the 3D model is known as well and for each point of the 3D model a quadruple is created of the form:

$$P_i = (X_i, Y_i, Z_i, T_i), \quad (4)$$

where P_i is a point belonging to the 3D model, X_i, Y_i and Z_i are its 3D coordinates and T_i the temperature value associated with the point. Then, quantitative measurements are done on the set of P_i points in order to measure the fiber orientation of a point on the surface of the complex shaped part. Results are presented next.

4. Experiments and Results

Seven specific points were chosen strategically on different regions on the part's surface to test the proposed solution. Points on strands with different fiber orientation were considered, i.e. how the strand was oriented after the part was moulded and consolidated. The surface geometric nature where the strand was located was also considered, i.e. whether the strand was situated on a spherical surface, on a cylindrical surface, on an edge between a curve and flat

Table 1 - Parameters used in the experiment

Parameter	Value
Diode-laser frequency	805 nm
Beam power used	5W of 30W
Shooting duration	0.1 seconds
Spot size on plate's surface	~3 mm
Recording time	10 seconds
Image used to extract fiber orientation	0.26 seconds after beam had stopped

Table 2 - Carbon/PEEK thermal properties [19]

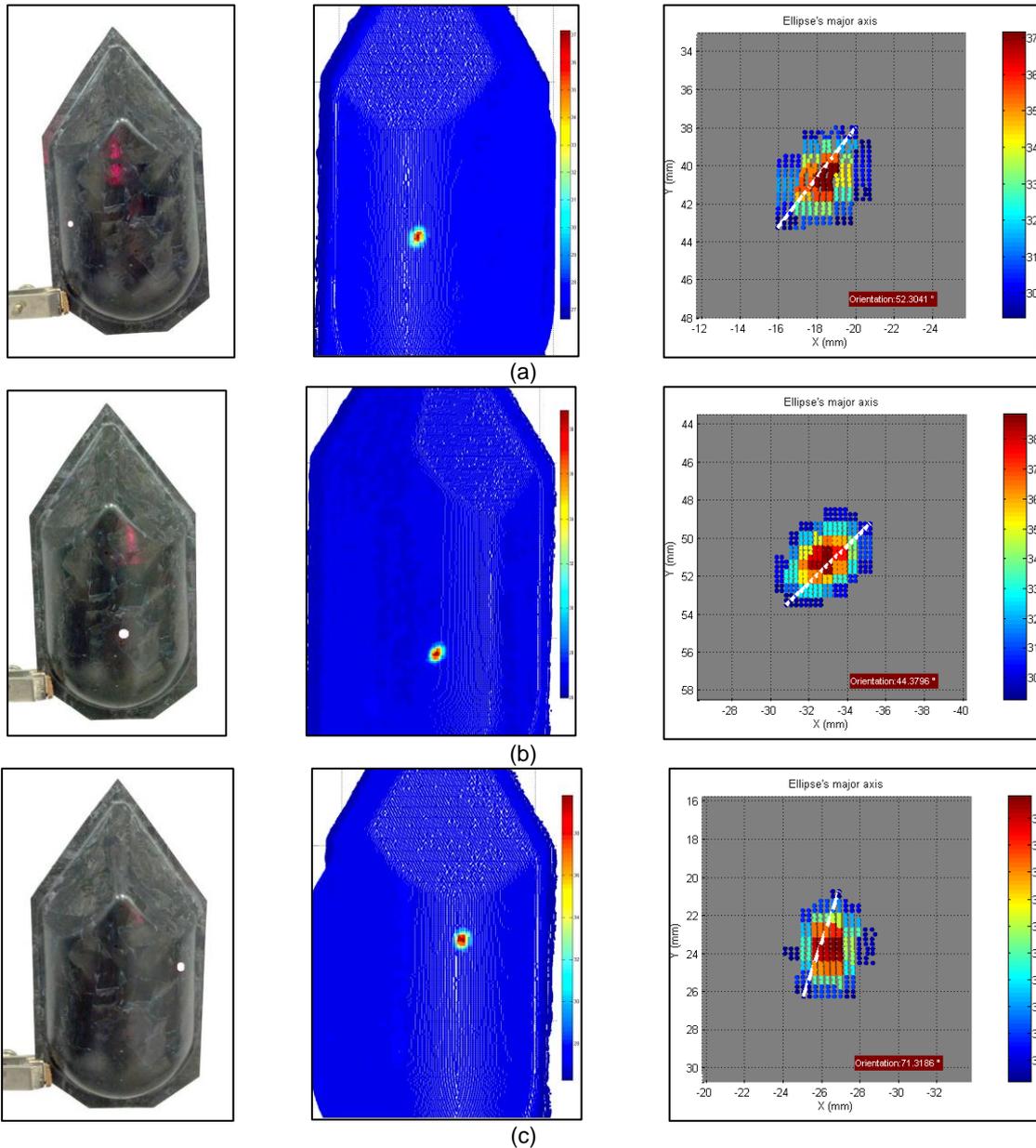
Parameter	Value
Thermal conductivity longitudinal to the fiber	$k_{\parallel} = 5.65 \left(\frac{W}{mK}\right)$
Thermal conductivity transverse to the fiber	$k_{\perp} = 0.335 \left(\frac{W}{mK}\right)$
Density	$\rho = 1584 \left(\frac{kg}{m^3}\right)$
Specific heat	$C = 1310 \left(\frac{J}{kg K}\right)$

surface, on a flat surface (but with a camera's angle of incidence greater than 0°) or on a corner (a region where three different surfaces meet).

For each one of the tested points, the position of the infrared camera did not change (it was neither rotated nor translated). Thus, the heating source had to be moved. For each test, the laser's optic fiber was repositioned so the desired point on the part could be heated. Another factor considered during the selection of the points to be tested was that the angle of incidence should not be greater than 60° (i.e. the angle between the camera optical axis and the normal to the point should be less than 60°). This restriction was also considered so that the quantitative analysis could be done with just one 2D infrared image (i.e. just one view).

The PTE (pulsed thermal ellipsometry) technique was then applied on each one of the points. Experimental parameters such as heating power and pulse duration were maintained constant during all the tests. Then, the heating and cooling down process were recorded using a mid-wave infrared camera (MWIR). As mentioned before, the pattern formed on the plate's surface is elliptical in anisotropic materials, which is the case of Carbon/PEEK composites and the ellipse's major axis is related to the fiber orientation. Experimental common parameters are listed in Table 1 and Carbon/PEEK thermal properties are listed in Table 2.

For each PTE test, the approach described in Sec. 3.3 was conducted. Thus, from each experiment, a 3D dense point cloud was calculated and based on the surface temperature distribution on the part an elliptical pattern was identified using a threshold temperature. The fiber orientation associated with the tested point was later calculated based on each elliptical pattern. In order to perform this calculation, each 3D point cloud was rotated around axis x and/or y to ensure that the normal to the point was parallel to the camera's angle of incidence. In other words, to calculate the ellipse orientation, the model was rotated so the thermal pattern could be quantified as it was nearly on a flat surface.



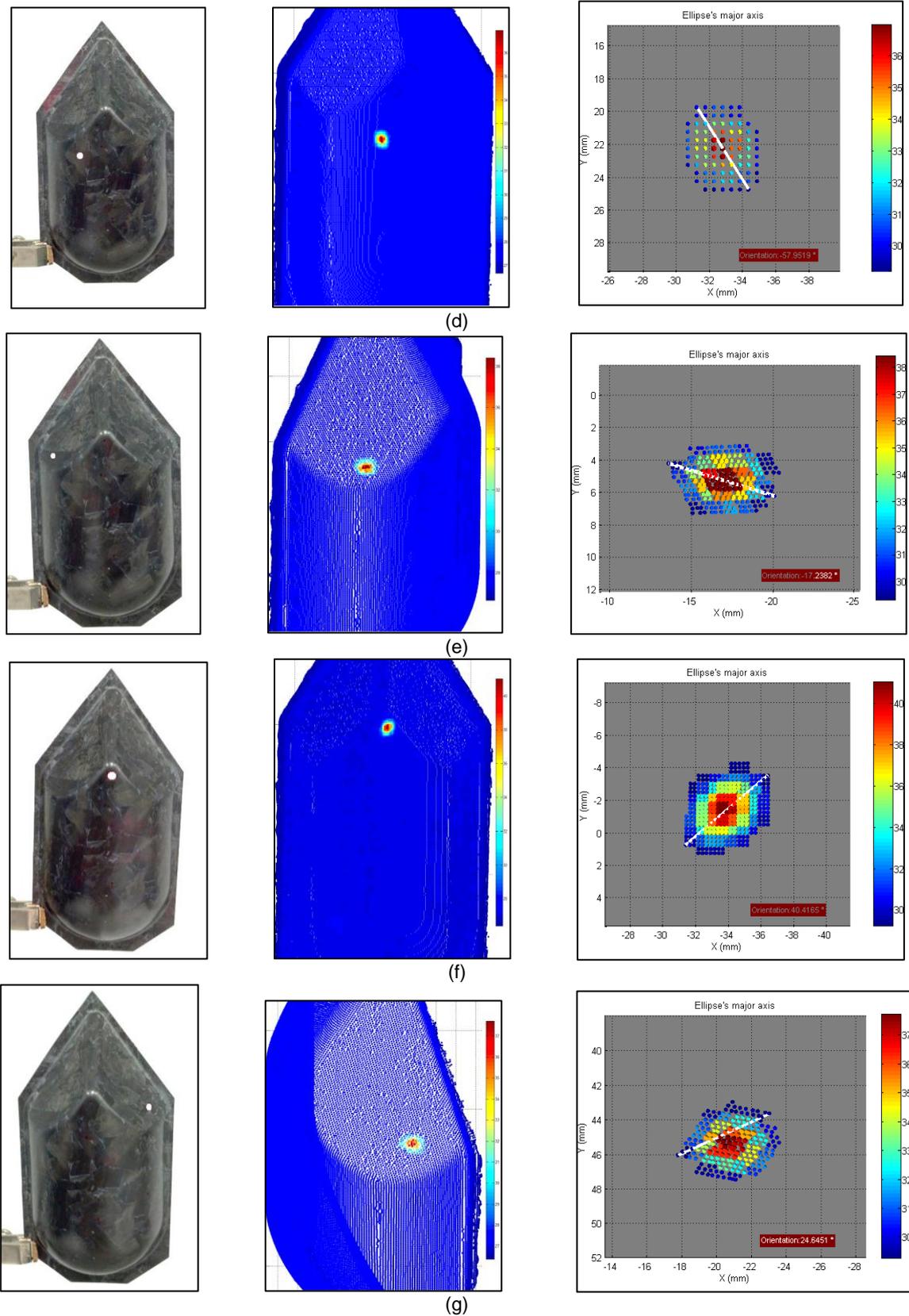


Figure 6 – Results obtained from the use of 3D and infrared information. (a-g) Points A through G. The left image is the visible image of the plate just before the experiment (the white spot indicates where was the heating spot), the central image shows part of the 3D model already rotated and the right image shows the threshold 3D points forming the ellipse and the white line indicates its major axis orientation .

Table 3 – Calculated fiber orientation on tested points

Tested point	Fiber orientation	Rotation angle (x,y)
A	52.30°	(0°,40.1°)
B	44.37°	(0°,-18.6°)
C	71.31°	(0°,-46.8°)
D	-57.95°	(0°,16.8°)
E	-17.23°	(30°,49°)
F	40.41°	(0°,-5°)
G	24.64°	(50°,-60°)

Figure 6 shows the images from 7 different tested spots. The left column shows the visible spectra image taken of the part just before the PTE experiment was carried out. The white point in these images indicates where the heating spot was located during the experiment. Images in the central column show a part of the 3D dense thermal point cloud rotated according to the values presented in Table 3 so the heating spot on the part was placed facing the view, i.e. the normal to the heating point is parallel to the camera's angle of incidence. Finally, the right column shows the elliptical thermal pattern identified from the rotated and thresholded 3D dense point cloud. Only the 3D points belonging to the ellipse are shown. The white line indicates the orientation of the ellipse's major axis which is, as discussed before, the same as the fiber orientation on that particular region.

Table 3 shows the fiber orientation result obtained for each tested point and also the angles used to calculate the final points rotation regarding axis x and axis y respectively that is applied before calculating the ellipse major axis orientation.

5. Conclusions

In this paper an infrared thermography technique using diode-laser to measure fiber orientation on CM was presented. PTE was applied in a complex shaped part and combined with a 3D model of the part's surface in order to conduct quantitative measurements of the fiber orientation on its surface. Fusion of 3D dense point cloud and thermal data was performed to enable the correct assessment of the fiber orientation in different regions of the complex shaped part without the distortion caused by the depth due to the geometric nature of the inspected part present when one uses only the 2D information.

Figure 7 shows the fiber orientation quantification using only 2D infrared images from a PTE experiment. Figure 7.a shows the 2D result of the tested point E (Figure 6.e) and Figure 7.b of the tested point G (Figure 6.g). The fiber orientation is completely different due to the above mentioned depth effect when using only the 2D information. When the 3D and thermal data are analysed together, the correct fiber orientation quantification is achieved. An observation of the part's color image (left column in Figure 6.e and Figure 6.g) and a careful analysis of the spot that was tested (indicated by the white point), reveals that the position/orientation of the strand indicates that the fiber orientation is in fact in the

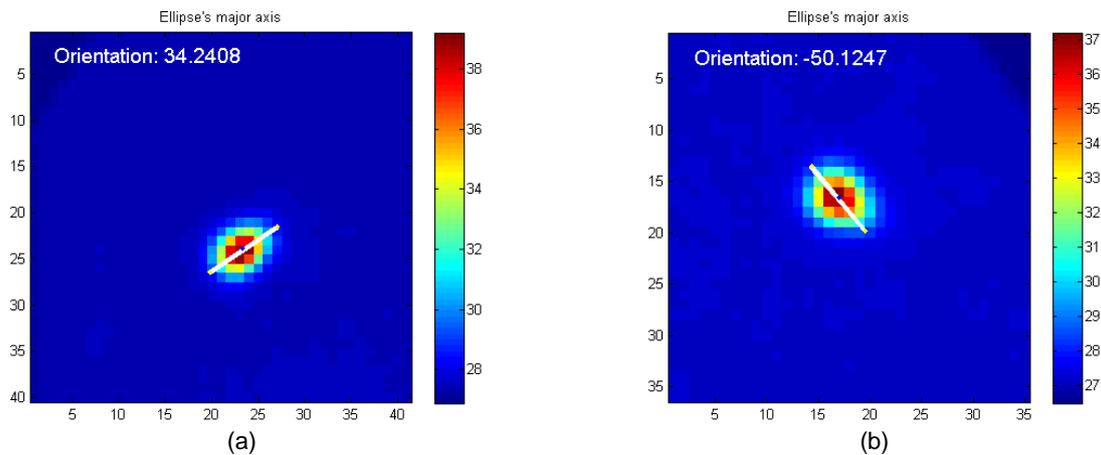


Figure 7 – Fiber orientation calculated using only 2D thermal images. (a) Point E. Same point tested in Figure 6.e. (b) Point G. Same point tested in Figure 6.g. The difference in the ellipse's major axis orientation is clearly observed when using fusion of 3D information and 2D infrared image and when using only 2D infrared image.

direction which was determined using the 3D information rather than the orientation calculated using only the 2D images. This shows how important it is to analyse not only the thermal information but also the 3D information when testing 3D complex shaped parts.

Different tested regions and respective results presented in Figure 6 show that the 3D information combined with the thermal information allow the fiber orientation measurement on complex 3D parts with only one field of view, i.e. with no need to reposition the part or the camera between two experiments for two different points. For example, Figure 6.a and Figure 6.c show accurate results of points where the camera's angle of incidence is almost 45° in modulus. Measurements on regions where there is an abrupt change on the surface geometric nature (e.g. edges (Figure 6.e) and corners (Figure 6.f)) are also possible.

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