# Development of a discontinuous finite element method to characterize vertical cracks using lock-in thermography

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### 1. Introduction

Detecting cracks in a nondestructive way is a challenge that has been addressed for decades but that is not completely solved yet. In the last decade infrared thermography has become the preferred photothermal technique to detect cracks, because of its capability to record surface temperature images. Several groups have applied IR thermography to the detection and characterization of cracks in different configurations (i.e. retrieving shape, size and thickness) [1-3].

In this work we calculate the surface temperature of an opaque sample with vertical cracks, when it is illuminated by a modulated and focused laser beam. In the case of infinite cracks analytical solutions have been found. For finite cracks, however, numerical calculations based on finite elements modeling have been performed. Conventional finite elements and finite differences fail when dealing with very thin cracks. To overcome this limitation we have developed a Discontinuous Galerkin (DG) finite element method. DG methods are natural tools to tackle physical problems with discontinuous solutions where classical finite element methods fail [4]. We expand the Bauman-Oden DG method, that was applied for the Laplace equation [5], in order to modelize the propagation of thermal waves in cracked materials. We show the non-obvious variational formulation of this DG method for modulated illuminations and lock-in infrared thermography.

From numerical simulations we have analyzed how the crack detection contrast depends on the laser spot size, on the distance to the crack and on the modulation frequency of the laser beam. Infrared thermography measurements performed on calibrated vertical cracks in metallic samples confirm the validity of the model.

## 2. Numerical simulations

The problem we are dealing with is depicted in Fig.1. An AISI-304 stainless steel sample contains a vertical crack located in plane y = 0. This opaque sample is illuminated by a modulated Gaussian laser of radius a, whose centre is at a distance d from the crack. Figure 2 shows the surface temperature profiles (amplitude and phase) along the y direction for an infinite vertical crack with a thermal resistance  $R = 10^3 \text{ m}^2 \text{ K-W}^{-1}$ . Calculations are performed with a laser beam of radius a = 0.5 mm, modulated at f = 0.8 Hz and d = 2 mm. As can be observed, both amplitude and phase undergo an abrupt jump when crossing the crack.

We introduce the crack contrast ( $\Delta$ ) that is defined as the amplitude (or phase) difference at both sides of the crack at the origin, normalized to the values at the center of the laser spot. The value of  $\Delta$  indicates the feasibility to detect and characterize a given crack. In Fig. 3 we show the evolution of the amplitude and phase contrast as a function of the thermal resistance. Calculations have been performed for a = 0.5 mm and f = 0.8 Hz. Two positions of the laser spot are analyzed. As can be observed the amplitude contrast increases with R until it reaches an asymptotic maximum. On the other hand, the maximum contrast is produced when the laser spot just touches the crack ( $d \approx a$ ). Regarding the phase contrast, it is positive when the spot is away from the crack, but it becomes negative when the laser spot overlaps a rather thick crack.

#### 3. Experimental results

To check the validity of predictions of section 2 we have performed lock-in thermography measurements performed in two AISI-304 samples separated by 25  $\mu$ m, simulating a crack with  $R = 10^3 \text{ m}^2$ K/W ( $R = L/K_{air}$ , where *L* is the thickness of the crack and  $K_{air}$  the thermal conductivity of air). Figure 4 shows the temperature profile along the *y* direction. The laser is modulated at f = 0.8 Hz with a = 1.0 mm and d = 0.8 mm. As can be observed the amplitude suddenly decreases when crossing the crack, but as expected from Fig. 3, the phase rises, i.e., the phase contrast is negative.

The final goal of this work is to retrieve the shape and location of the crack from surface thermograms. This is a severely ill-posed problem (i.e. crack reconstruction strongly depends on noise in the data) that requires rather sophisticated inversion procedures. Note that this method based on using discontinuous finite elements allows dealing

with cracks of finite size and even out of plane. The present work can be considered as the first step to reconstruct cracks of arbitrary shape and size from lock-in thermography. Moreover, numerical models developed for modulated excitation can be easily improved to work under pulsed excitation, opening the possibility of fast crack characterization.



Fig. 1. Scheme of the vertical crack on a semi-infinite sample.



**Fig. 2** Surface temperature profile (amplitude and phase) along the y direction for an infinite vertical crack with  $R = 10^3 m^2 K/W$ . Calculations are performed for AISI-304 at f = 0.8 Hz, using a laser beam of radius 0.5 mm, d = 2 mm.



**Fig. 3.** Amplitude and phase contrast as a function of thermal resistance, for an AISI-304 sample excited by a modulated (f = 0.8 Hz) laser of radius 0.5 mm. Continuous line stands for d = 1 mm and dotted line for d = 0.3 mm.



**Fig. 4.** Amplitude and phase temperature profiles along the y direction for an AISI-304 sample with an infinite crack 25  $\mu$ m thick (R = 10<sup>-3</sup> m<sup>2</sup>K/W) measured at 0.8 Hz.

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