

# Dimensionless numerical sensitivity analysis of narrow cracks by means of infrared lock-in thermography

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## Abstract

Lock-in infrared thermography has been identified as a highly suitable technique for a quantitative defect characterization. In this work, first, a complete dimensionless reformulation of the thermographic investigation is provided. As a consequence, the constraints of particular experimental setups or material properties can be overcome preserving the full physical information of the experiment. The resulting model has been numerically solved and successfully validated by using experimental thermographic data. Second, the developed dimensionless model has been used as input for a global sensitivity analysis. Overall, the obtained results provide an experimental guideline for an optimized thermographic defect characterization.

## 1 Introduction



**Fig. 1.** Schematic view of a laser-spot lockin IR experiment with a focused continuous wave (CW) laser beam optical excitation.

The quantitative non-destructive search of defects in materials, such as cracks or delaminations, is a challenge in many industrial fields. In this regards, laser-spot lock-in thermography is a successful alternative for a non-intrusive, safe and accurate characterization of quasi-superficial defects due to an enhanced signal-to-noise ratio. As a consequence, very small defects, in the order of microns, can successfully be measured.

However, laboratory measurements must be complemented with theoretical calculations in order to obtain a fully quantitative geometrical characterization of the involved defects. In this line, aiming to extend the laser-spot lock-in thermography capabilities to an unconstrained potential in terms of material thermal properties, experimental conditions or involved defect geometrical implications, in this work, a general dimensionless numerical model used for crack thermographic quantitative characterization is provided.

After experimental validation over calibrated cracked samples, the numerical model has been used to feed an stochastic global sensitivity analysis (GSA) able to determine the thermographic detection limits and optimal parametric set-ups for maximized crack detection capabilities in terms of sensitivity and cross parametric correlations.

#### 2. Dimensionless formulation

Laser-spot lock-in thermography is, in essence, a heat diffusion problem guided by a harmonic heating boundary condition, implemented in our particular case, by a Gaussian profile optical source whereas the non illuminated faces of the sample are assumed adiabatic. The crack, introduced in the numerical discretization as a 2D planar domain, is modeled assuming heat flux continuity through it and as a thermal resistance,  $R_{th} = \frac{w}{\kappa_{air}}$ , of width w filled by air with a thermal conductivity  $\kappa_{air}$ . In the resulting set of equations (1), f is the modulation frequency,  $\kappa$  the thermal conductivity of the material,  $\eta$  the power fraction absorbed by the sample, P the power of the illuminating source centered at (0,  $y_0$ , 0) focused to a radius  $r_g$  and [] is the jump operator.

In order to remove the dimensions of (1), characteristic length  $L_c$ , time  $t_c$  and temperature  $T_c$  scales must be introduced as the thermal diffusion length ( $\mu$ ), the inverse of the modulation frequency and the magnitude removing the dimensions from Fourier equation, shown by Eqs. (3)-(5) respectively. This selection leads to the dimensionless sort of equations (2) where the barred variables refer to dimensionless quantities,  $\Pi_1 \equiv 2(\mu/r_g)^2$ ,  $\Pi_2 \equiv \kappa R_{th}/\mu$ .

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$$\begin{cases} \alpha \nabla^2 T = \frac{\partial T}{\partial t} \\ -\kappa \nabla T|_{\text{illuminated}} = \frac{2P\eta}{\pi r_g^2} e^{-2\left[\left(\frac{x}{r_g}\right)^2 + \left(\frac{y-y_0}{r_g}\right)^2\right]} \cos(2\pi f t) \\ -\kappa \nabla T|_{\text{non-illuminated}} = 0 \\ \|-\kappa \nabla T\||_{\text{defect}} = 0 \\ \Delta T|_{\text{defect}} = \kappa R_{th} \nabla T \end{cases} \tag{1}$$

$$\begin{cases} \bar{\nabla}^2 \bar{T} = \frac{\partial \bar{T}}{\partial t} \\ \bar{\nabla} \bar{T}|_{\text{illuminated}} = \Pi_1 e^{-\Pi_1 \left[\bar{x}^2 + (\bar{y} - \Pi_6)^2\right]} \cos(2\bar{t}) \\ \bar{\nabla} \bar{T}|_{\text{inon-illuminated}} = 0 \\ \|\bar{\nabla} \bar{T}\||_{\text{defect}} = 0 \\ \Delta \bar{T}|_{\text{defect}} = \pi R_{th} \nabla T \end{cases} \tag{2}$$

As can be seen in the set of Eqs. (2) this model does not imply any geometrical restriction for the defect. However, for practical purposes, this work has been carried out considering planar open-surface cracks. As a consequence, the crack is fully characterized by two spatial parameters, length (l) and depth (d), and the inclination angle  $(\theta)$  measured from the illuminated side  $(\bar{y} > 0)$  of the sample. According to the selected characteristic length scale the two spatial parameters must be rescaled as  $\Pi_3 \equiv l/\mu$  and  $\Pi_4 \equiv d/\mu$ . In order to maintain the same notation for all the dimensionless parameters,  $\Pi_5 \equiv \theta$  and  $\Pi_6 \equiv \bar{y}_0$ .

$$L_c \equiv \mu \equiv \sqrt{\frac{\alpha}{\pi f}}$$
 (3)  $t_c \equiv \frac{1}{2\pi f}$  (4)  $T_c \equiv \frac{P\eta}{\mu\kappa}$  (5)

## 3. Global sensitivity analysis (GSA)

After experimentally validating the proposed dimensionless numerical model, it has been used as input to a global sensitivity analysis. For an appropriate statistical approach, the probe space has been stochastically sampled on 1000 test cases by using the Latin Hypercube Sampling (LHS) algorithm. As a result, the global sensitivity and correlations between the proposed dimensionless parameters together with their impact on the thermographic results are quantitatively determined by means of the partial correlation coefficient (p), shown in Fig.2. In addition, the model response as a function of each parameter value is also computed selecting the maximum sensitivity regions. This analysis has allowed to identify the dimensionless sensitivity parametric ranges and the corresponding predominancy of each parameter on the selected model response.



Fig. 2. Partial correlation coefficient (p) distributions for each dimensionless parameter as a function of the spatial coordinate.

Overall, the obtained dimensionless sensitivity results reveal that laser-spot lock-in infrared thermography is an outstanding experimental technique when dealing with a quantitative analysis of narrow or small cracks, in the order of microns, since the computed sensitivities show a clear increase for decreasing crack length, depth and width values. Moreover, the provided quantitative dimensionless parametric sensitivity ranges can be used as a general experimental guideline for optimal lock-in thermographic investigation of narrow open-surface cracks in materials.

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